Physics of Fluids (NAPF-12)
April 8, 2014, 9.00 – 12.00.

This is a closed-book exam; only simple pocket calculators are allowed. There are 4 questions. Every question should be made on a separate sheet. Put your name and s-number on every sheet that you hand in. Also return the sheet with the exam questions.

Question 1 (total 18 p)

In the Eulerian description of fluid motion, three types of curves (flow lines) are commonly used to describe fluid motion: streamlines, path lines and streak lines.

4p a) Give a description of each of these three flow lines. Use drawings, if needed, to support your answer.

Using experimental techniques the velocity of a two-dimensional flow is determined for every point in space (x, y) and instant in time t to be: $\boldsymbol{u} = (u_0, v_0 \sin(\omega t - kx))$, with u_0 and v_0 reference velocities, ω a characteristic angular frequency and $1/k$ a characteristic wavelength.

6p b) Determine the equation of the streamline that passes through the origin at $t = 0$.

6p c) Determine the equation of the path line that passes through the origin at $t = 0$.

2p d) For what kind of flows do the streamlines, path lines and streak lines coincide? For which special case of the flow analyzed above does this happen?

Question 2 (total 12 p)

A proposed conservation law for ξ, a new fluid property, takes the following form:

$$\frac{d}{dt}\int_{V(t)} \xi\rho dV + \int_{A(t)} \mathbf{\Theta}\cdot\mathbf{n}\, dA = 0,$$

where $V(t)$ is a material volume that moves with the fluid velocity $\boldsymbol{u}$, $A(t)$ is the surface of $V(t)$, $\mathbf{n}$ is the outward normal at the surface, ρ is the fluid density, and $\mathbf{\Theta} = -\rho\,\nabla\xi$.

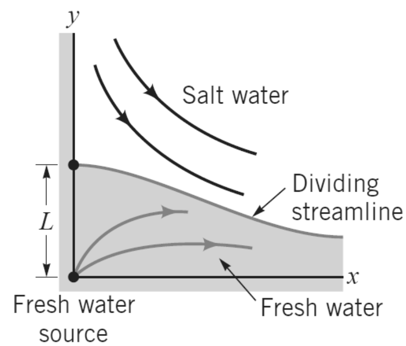
Use Reynolds transport theorem, Gauss divergence theorem and the continuity equation, respectively, to show that the partial differential equation

$$\frac{\partial\xi}{\partial t} + \mathbf{u}\cdot\nabla\xi = \frac{1}{\rho}\nabla\cdot\left(\rho\nabla\xi\right)$$

corresponds to the proposed conservation law.

Question 3 (total 24 p)

At a certain point at the beach the coast line makes a right-angle bend. A fresh-water reservoir is located in the corner (see the figure below). The water has density ρ and



viscosity μ. By means of ideal flow theory, this two-dimensional problem can be analyzed using the superposition of a corner flow (with parameter A) and a source of strength m in terms of the following velocity potential $\phi(x,y)$:

$$\phi = A\left(x^2 - y^2\right) + \frac{m}{2\pi}\ln\sqrt{x^2 + y^2}.$$

6p a) Show that the flow described by ϕ is incompressible and irrotational.

6p b) Rewrite $\phi(x,y)$ in terms of polar coordinates (r,θ) and use this to derive the corresponding stream function:

$$\psi = Ar^2\sin(2\theta) + \frac{m}{2\pi}\theta.$$

$$[u_r = \frac{\partial\phi}{\partial r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta}, \quad u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{\partial\psi}{\partial r},$$

$$\sin(2x) = 2\sin x\cos x, \ \cos(2x) = \cos^2(x) - \sin^2(x)]$$

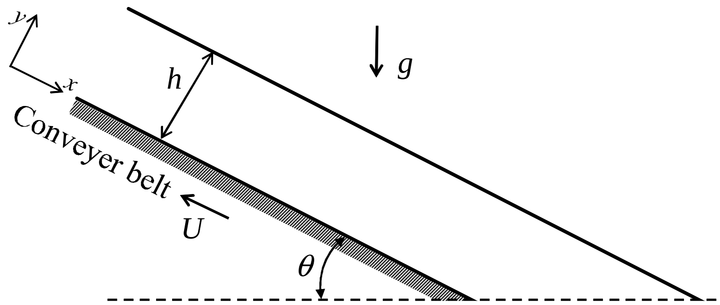
5p c) Determine the location of the stagnation point (in Cartesian or polar coordinates). Write your answer in terms of A and m.

2p d) Show that the value of the stream function on the dividing stream line (see figure) is equal to $m/4$.

5p e) Finally, determine the pressure difference between point $(x, y) = (L, 0)$ and point $(x, y) = (0, L)$. Write your answer in terms of A and L.

Question 4 (total 22 p)

A wide conveyer belt passes through a container of a viscous liquid (with density ρ and viscosity μ). The belt moves up under an angle θ with a constant velocity U. Because of viscous forces the belt picks up a film of fluid of thickness h. Gravity



tends to make the fluid drain down the belt. Assume that the flow in the fluid layer is incompressible, laminar, steady, and fully developed. The presence of atmospheric pressure at the free surface ensures that $\partial p/\partial x = 0$.

4p a) Write down the continuity equation and use this to show that the components of the velocity vector with respect to the Cartesian axes (x,y), reduce to $u = u(y)$ and $v = 0$.

12p b) Write down the Navier-Stokes momentum equations and solve for the velocity $u(y)$. Calculate the volume flow rate per unit out-of-plane width and the average film velocity.

6p c) For a given angle θ the velocity U has to be adjusted to ensure that no fluid is drained downwards. Calculate the minimal velocity $U_{\min}$ needed to ensure that no fluid is drained back into the container. Discuss the dependence of $U_{\min}$ on ρ and μ.

N.B. For all questions: when for some reason you are unable to answer a part of a question (a, b, etc.), make a realistic assumption and use this for the rest of the question.

GOOD LUCK!